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CP-Violating Lepton-Energy Correlation in $e\bar{e} \to t\bar{t}$

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ABSTRACT

In order to observe a signal of possible CP violation in top-quark couplings, we have studied energy correlation of the final leptons in $e^+e^- \to t\bar{t} \to \ell^+\ell^- X \,/\, \ell^\pm X$ at future linear colliders. Applying the recently-proposed optimal method, we have compared the statistical significances of CP-violation-parameter determination using double- and single-lepton distributions. We have found that the single-lepton-distribution analysis is more advantageous.

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The top quark, thanks to its huge mass, is expected to provide us a good opportunity to study beyond-the-Standard-Model physics. Indeed, as many authors pointed [$1-8$], CP violation in its production and decay could be a useful signal for possible non-standard interactions. This is because (*i*) the CP violation in the top-quark couplings induced within the SM is far negligible and (*ii*) a lot of information on the top quark is to be transferred to the secondary leptons without getting obscured by the hadronization effects.

In a recent paper, we have investigated CP violation in the $t\bar{t}$-pair productions and their subsequent decays at next linear colliders (NLC) [8]. We have focused there on the single-lepton-energy distributions. In this note, we study both the double- and single-lepton-energy distributions in the process $e^+e^- \to t\bar{t} \to \ell^+\ell^- X \,/\, \ell^\pm X$, and we compare the expected precision of CP-violation-parameter determination in each case. For this purpose, we apply the recently-proposed optimal procedure [9].

Let us briefly summarize the main points of this method first. Suppose we have a cross section

$$\frac{d\sigma}{d\phi}(\equiv \mathit{\Sigma}(\phi)) = \sum_i c_i f_i(\phi)$$

where the $f_i(\phi)$ are known functions of the location in final-state phase space ϕ and the c_i are model-dependent coefficients. The goal would be to determine c_i's. It can be done by using appropriate weighting functions $w_i(\phi)$ such that $\int w_i(\phi)\Sigma(\phi)d\phi = c_i$. Generally, different choices for $w_i(\phi)$ are possible, but there is a unique choice such that the resultant statistical error is minimized. Such functions are given by

$$w_i(\phi) = \sum_j X_{ij} f_j(\phi)/\mathit{\Sigma}(\phi)\,, \tag{1}$$

where X_{ij} is the inverse matrix of M_{ij} which is defined as

$$M_{ij} \equiv \int \frac{f_i(\phi)f_j(\phi)}{\mathit{\Sigma}(\phi)} d\phi\,. \tag{2}$$

When we take these weighting functions, the statistical uncertainty of c_i becomes

$$\Delta c_i = \sqrt{X_{ii}\, \sigma_T/N}\,, \tag{3}$$

where $\sigma_T \equiv \int (d\sigma/d\phi)d\phi$ and $N = L_{\rm eff}\sigma_T$ is the total number of events, with $L_{\rm eff}$ being the integrated luminosity times efficiency.

In our analyses, we assume that only interactions of the third generation of quarks may be affected by beyond-the-Standard-Model physics and that all non-standard effects in the production process ($e^+e^- \to t\bar{t}$) can be represented by the photon and Z-boson exchange in the s-channel. The effective $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices are parameterized in the following form

$$\Gamma^\mu = \frac{g}{2}\bar{u}(p_t)\Big[\,\gamma^\mu(A_v - B_v\gamma_5) + \frac{(p_t - p_{\bar{t}})^\mu}{2m_t}(C_v - D_v\gamma_5)\,\Big]v(p_{\bar{t}}), \tag{4}$$

$$(v = \gamma \text{ or } Z)$$

where g is the SU(2) gauge-coupling constant. In principle, there are also four-Fermi operators which may contribute to the process of $t\bar{t}$ production. However, as it has been verified in Ref. [10], their net effect is equivalent to a modification of A_v and B_v. Therefore, without loosing generality we may restrict ourself to the vertex corrections only.

For the on-shell W, we will adopt the following parameterization of the tbW vertex:

$$\Gamma^\mu = -\frac{g}{\sqrt{2}}V_{tb}\,\bar{u}(p_b)\Big[\,\gamma^\mu(f_1^L P_L + f_1^R P_R) - \frac{i\sigma^{\mu\nu}k_\nu}{M_W}(f_2^L P_L + f_2^R P_R)\,\Big]u(p_t), \tag{5}$$

$$\bar{\Gamma}^\mu = -\frac{g}{\sqrt{2}}V_{tb}^*\,\bar{v}(p_t)\Big[\,\gamma^\mu(\bar{f}_1^L P_L + \bar{f}_1^R P_R) - \frac{i\sigma^{\mu\nu}k_\nu}{M_W}(\bar{f}_2^L P_L + \bar{f}_2^R P_R)\,\Big]v(p_b), \tag{6}$$

where $P_{L/R} \equiv (1 \mp \gamma_5)/2$, V_{tb} is the (tb) element of the Kobayashi-Maskawa matrix and k is W's momentum.

Using the above parameterization, applying the narrow-width approximation for the decaying intermediate particles, and assuming that the Standard-Model contribution dominates the CP-conserving part, we get the following normalized

double- and single-lepton-energy distributions of the reduced lepton energy $\overset{(-)}{x} \equiv 2E\sqrt{(1-\beta)/(1+\beta)}/m_t$, E being the energy of $\ell^{\pm}$ in the e^+e^- c.m. system, and $\beta \equiv \sqrt{1-4m_t^2/s}$:

Double distribution

$$\frac{1}{\sigma}\frac{d^2\sigma}{dx\, d\bar{x}} = \sum_{i=1}^{3} c_i f_i(x,\bar{x}), \tag{7}$$

where x and $\bar{x}$ are for ℓ^+ and ℓ^- respectively,

$$c_1 = 1, \quad c_2 = \xi, \quad c_3 = \frac{1}{2}\mathrm{Re}(f_2^R - \bar{f}_2^L)$$

and

$$\begin{aligned}
f_1(x,\bar{x}) &= f(x)f(\bar{x}) + \eta'\, g(x)g(\bar{x}) + \eta\,[\, f(x)g(\bar{x}) + g(x)f(\bar{x})\,],\\
f_2(x,\bar{x}) &= f(x)g(\bar{x}) - g(x)f(\bar{x}),\\
f_3(x,\bar{x}) &= \delta f(x)f(\bar{x}) - f(x)\delta f(\bar{x}) + \eta'\,[\,\delta g(x)g(\bar{x}) - g(x)\delta g(\bar{x})\,]\\
&\qquad +\eta\,[\,\delta f(x)g(\bar{x}) - f(x)\delta g(\bar{x}) + \delta g(x)f(\bar{x}) - g(x)\delta f(\bar{x})\,].
\end{aligned}$$

Single Distribution

$$\frac{1}{\sigma^{\pm}}\frac{d\sigma^{\pm}}{dx} = \sum_{i=1}^{3} c_i^{\pm} f_i(x), \tag{8}$$

where $\pm$ corresponds to $\ell^{\pm}$,

$$c_1^{\pm} = 1, \quad c_2^{\pm} = \mp\xi, \quad c_3^{+} = \mathrm{Re}(f_2^R), \quad c_3^{-} = \mathrm{Re}(\bar{f}_2^L)$$

and

$$f_1(x) = f(x) + \eta\, g(x), \quad f_2(x) = g(x), \quad f_3(x) = \delta f(x) + \eta\, \delta g(x).$$

Since all the functions and parameters in these formulas are to be found in Refs.[7, 8], we only remind here the normalization of $f(x)$, $\delta f(x)$, $g(x)$ and $\delta g(x)$:

$$\int f(x)dx = 1, \quad \int \delta f(x)dx = \int g(x)dx = \int \delta g(x)dx = 0. \tag{9}$$

η, η' and ξ are numerically given at $\sqrt{s} = 500$ GeV as

$$\eta = 0.2021, \quad \eta' = 1.3034, \quad \xi = -1.0572\,\mathrm{Re}(D_\gamma) - 0.1771\,\mathrm{Re}(D_Z)$$

for the SM parameters $\sin^2\theta_W = 0.2325$, $M_W = 80.26$ GeV, $M_Z = 91.1884$ GeV, $\mathit{\Gamma}_Z = 2.4963$ GeV and $m_t = 180$ GeV.

In Eqs.(7,8), CP is violated by non-vanishing ξ and/or $\mathrm{Re}(f_2^R - \bar{f}_2^L)$ terms.[♯1] First, let us discuss how to observe a combined signal of CP violation emerging via both of these parameters. The energy-spectrum asymmetry $a(x)$ defined as

$$a(x) \equiv \frac{d\sigma^-/dx - d\sigma^+/dx}{d\sigma^-/dx + d\sigma^+/dx}$$

has been found as a useful measure of CP violation via ξ [4, 7]. In Ref.[8] we have computed $a(x)$ for the case where both ξ and $\mathrm{Re}(f_2^R - \bar{f}_2^L)$ terms exist. Practically however, measuring differential asymmetries like $a(x)$ is a challenging task since they are not integrated and therefore expected statistics cannot be high. For this reason, we shall discuss another observable here.

A possible asymmetry would be for instance

$$A_{\ell\ell} \equiv \frac{\displaystyle\int\!\!\int_{x<\bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}} - \int\!\!\int_{x>\bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}}}{\displaystyle\int\!\!\int_{x<\bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}} + \int\!\!\int_{x>\bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}}}\,. \tag{10}$$

For our SM parameters, it becomes

$$\begin{aligned} A_{\ell\ell} &= 0.3638\,\mathrm{Re}(D_\gamma) + 0.0609\,\mathrm{Re}(D_Z) + 0.3089\,\mathrm{Re}(f_2^R - \bar{f}_2^L) \\ &= -0.3441\,\xi + 0.3089\,\mathrm{Re}(f_2^R - \bar{f}_2^L). \end{aligned} \tag{11}$$

For $\mathrm{Re}(D_\gamma) = \mathrm{Re}(D_Z) = \mathrm{Re}(f_2^R) = -\mathrm{Re}(\bar{f}_2^L) = 0.2$, e.g., we have

$$A_{\ell\ell} = 0.2085$$

[♯1] In the present note, t, $\bar{t}$ and $W^\pm$ are assumed to be on their mass shell since we are adopting the narrow-width approximation for them, and the contribution from the imaginary part of the Z propagator is also negligible since s is much larger than M_Z^2. Therefore we do not have to consider CP-violating effects triggered by the interference of the propagators of those unstable particles with any other non-standard terms [11].

and its statistical error is estimated to be

$$\Delta A_{\ell\ell} = \sqrt{(1-A_{\ell\ell}^2)/N_{\ell\ell}} = 0.9780/\sqrt{N_{\ell\ell}}.$$

Since $\sigma_{e\bar{e}\to t\bar{t}} = 0.60$ pb for $\sqrt{s} = 500$ GeV, the expected number of events is $N_{\ell\ell} = 600\,\epsilon_{\ell\ell} L B_\ell^2$, where $\epsilon_{\ell\ell}$ stands for the $\ell^+\ell^-$ tagging efficiency ($= \epsilon_\ell^2$; ϵ_ℓ is the single-lepton-detection efficiency), L is the integrated luminosity in fb^{-1} unit, and $B_\ell(\simeq 0.22)$ is the leptonic branching ratio for t. Consequently we obtain the following result for the error

$$\Delta A_{\ell\ell} = 0.1815/\sqrt{\epsilon_{\ell\ell} L}, \tag{12}$$

and thereby we are able to compute the statistical significance of the asymmetry observation $N_{SD} = |A_{\ell\ell}|/\Delta A_{\ell\ell}$.

In Fig.1 we present lines of constant N_{SD} as functions of $\mathrm{Re}(D_\gamma) = \mathrm{Re}(D_Z)$ and $\mathrm{Re}(f_2^R - \bar{f}_2^L)$ for $L = 50$ fb^{-1} and $\epsilon_{\ell\ell} = 0.5$ (which mean $N_{\ell\ell} = 726$). Two solid lines, dashed lines and dotted lines are determined by

$$|\, 0.4247\,\mathrm{Re}(D_{\gamma,Z}) + 0.3089\,\mathrm{Re}(f_2^R - \bar{f}_2^L)\,| = N_{SD}/\sqrt{N_{SD}^2 + N_{\ell\ell}}$$

for N_{SD} =1, 2 and 3 respectively. We can confirm $A_{\ell\ell}$ to be non-zero at 1σ, 2σ and 3σ level when the parameters are outside the corresponding lines. It can be seen that we have good chances for observing the effect at future NLC unless there is a conspiracy cancellation between those parameters. Table 1 shows the $\sqrt{s}$ dependence of N_{SD} for the same $\epsilon_{\ell\ell} L$.

In order to discover the mechanism of CP violation, however, it is indispensable to separate the parameter in the top-quark production (ξ)[♯2] and that in the decay ($\mathrm{Re}(f_2^R - \bar{f}_2^L)$). We shall apply the optimal procedure of Ref.[9] to the double distribution first. Using the functions in Eq.(7), we may calculate elements of the matrix M and X defined in Eqs.(1, 2):

$$M_{11} = 1, \quad M_{12} = M_{13} = 0, \quad M_{22} = 0.2070, \quad M_{23} = -0.3368, \quad M_{33} = 0.6049$$

[♯2] We use ξ instead of $\mathrm{Re}(D_{\gamma,Z})$ as a basic parameter when we discuss parameter measurements, since ξ is directly related to the distributions Eqs.(7) and (8).

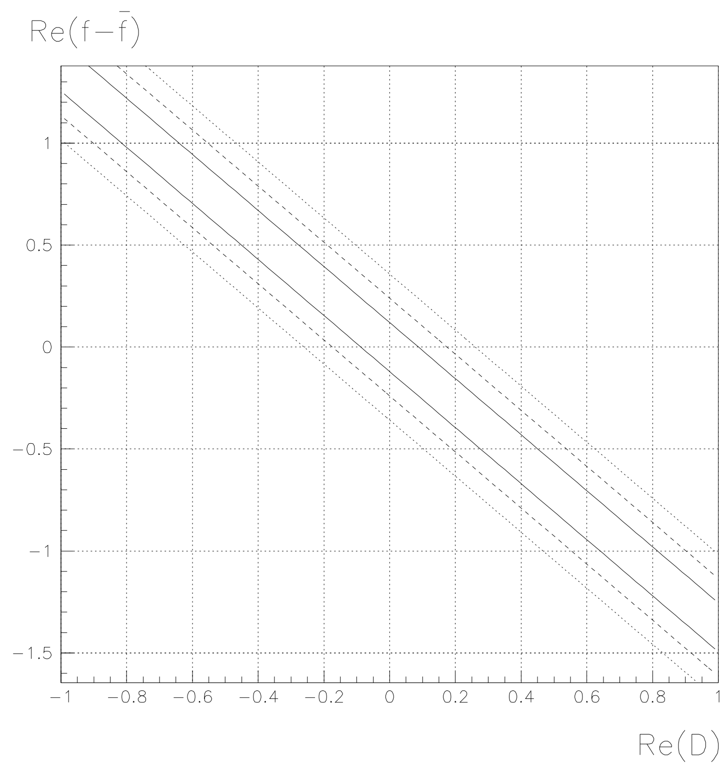

Figure 1: We can confirm the asymmetry $A_{\ell\ell}$ to be non-zero at 1σ, 2σ and 3σ level when the parameters $\mathrm{Re}(D_{\gamma,Z})$ and $\mathrm{Re}(f_2^R - \bar{f}_2^L)$ are outside the two solid lines, dashed lines and dotted lines respectively.

and

$$X_{11} = 1, \quad X_{12} = X_{13} = 0, \quad X_{22} = 51.3389, \quad X_{23} = 28.5825, \quad X_{33} = 17.5662.$$

This means the parameters are measured with errors of [♯3]

$$\Delta\xi = 7.1651/\sqrt{N_{\ell\ell}}, \quad \Delta\mathrm{Re}(f_2^R - \bar{f}_2^L)(= 2\sqrt{X_{22}/N_{\ell\ell}}) = 8.3824/\sqrt{N_{\ell\ell}}. \qquad (13)$$

Next we shall consider what we can gain from the single distribution. We

[♯3] Note that σ_T in Eq.(3) is unity in our case since we are using normalized distributions.

$\sqrt{s}$ (GeV)	500	600	700	800	900	1000
$\sigma_{e\bar{e}\to t\bar{t}}$ (pb)	0.60	0.44	0.33	0.25	0.20	0.16
$P=0.1$	2.8	2.5	2.3	2.0	1.8	1.7
	(0.1043)	(0.1097)	(0.1132)	(0.1155)	(0.1171)	(0.1183)
$P=0.2$	5.7	5.2	4.6	4.1	3.7	3.4
	(0.2085)	(0.2195)	(0.2263)	(0.2309)	(0.2342)	(0.2365)
$P=0.3$	8.9	8.0	7.2	6.4	5.8	5.3
	(0.3127)	(0.3292)	(0.3395)	(0.3464)	(0.3513)	(0.3548)
$P=0.4$	12.4	11.3	10.1	9.1	8.2	7.5
	(0.4170)	(0.4389)	(0.4527)	(0.4619)	(0.4683)	(0.4730)

Table 1: Energy dependence of the statistical significance N_{SD} of $A_{\ell\ell}$ measurement for CP-violating parameters $\mathrm{Re}(D_\gamma) = \mathrm{Re}(D_Z) = \mathrm{Re}(f_2^R) = -\mathrm{Re}(\bar{f}_2^L)(\equiv P)$ =0.1, 0.2, 0.3 and 0.4. The numbers below N_{SD} (those in the parentheses) are for the asymmetry $A_{\ell\ell}$.

have from Eq.(8)

$$M_{11}=1,\quad M_{12}=M_{13}=0,\quad M_{22}=0.0898,\quad M_{23}=0.1499,\quad M_{33}=0.2699$$

and

$$X_{11}=1,\quad X_{12}=X_{13}=0,\quad X_{22}=151.9915,\quad X_{23}=-84.4279,\quad X_{33}=50.6035.$$

Therefore we get $\Delta\xi = 12.3285/\sqrt{N_\ell}$ and $\Delta\mathrm{Re}(f_2^R) = 7.1136/\sqrt{N_\ell}$ from the ℓ^+ distribution, and analogous for $\Delta\xi$ and $\Delta\mathrm{Re}(\bar{f}_2^L)$ from the ℓ^- distribution. Since these two distributions are statistically independent, we can combine them as

$$\Delta\xi = 8.7176/\sqrt{N_\ell},\quad \Delta\mathrm{Re}(f_2^R - \bar{f}_2^L) = 10.0601/\sqrt{N_\ell}\,. \tag{14}$$

It is premature to conclude from Eqs.(13) and (14) that we get a better precision in the analysis with the double distribution. As it could be observed in the numerators in Eqs.(13, 14), *we lose some information when integrating the double distribution on one variable.* However, *the size of the expected uncertainty*

depends also on the number of events. That is, $N_{\ell\ell}$ is suppressed by the extra factor $\epsilon_\ell B_\ell$ comparing to N_ℓ. This suppression is crucial even if we could achieve $\epsilon_\ell = 1$. For N pairs of $t\bar{t}$ and $\epsilon_\ell = 1$ we obtain

$$\Delta\xi = 32.5686/\sqrt{N}, \quad \Delta\mathrm{Re}(f_2^R - \bar{f}_2^L) = 38.1018/\sqrt{N}$$

from the double distribution, while

$$\Delta\xi = 18.5859/\sqrt{N}, \quad \Delta\mathrm{Re}(f_2^R - \bar{f}_2^L) = 21.4484/\sqrt{N}$$

from the single distribution.[♯4] Therefore we may say that the single-lepton-distribution analysis is more advantageous for measuring the parameters individually.

In summary, we have studied how to observe possible CP violation in $e^+e^- \to t\bar{t} \to \ell^+\ell^- X$ and $\ell^\pm X$ at NLC. For this purpose, CP-violating distributions of the final-lepton energies are very useful. Using these quantities, we introduced a new asymmetry $A_{\ell\ell}$ in Eq.(10), which was shown to be effective. Then, applying the optimal procedure [9], we computed the statistical significances of CP-violation-parameter determination in analyses with the double- and single-lepton-energy distributions. Taking into account the size of the leptonic branching ratio of the top quark and its detection efficiency, we conclude that the use of the single-lepton distribution is more advantageous to determine each CP-violation parameter separately.

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[♯4] If we take $\epsilon_\ell B_\ell$ =0.15 as a more realistic value [12], we are led to the same results as in [8].

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